FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

WIRTSCHAFTS MATHEMATIK

Master Thesis:

Additional information for compressed sensing with applications in computed tomography

Task

A common challenge in computed tomography (CT), i.e., projection-based tomography using X-rays (CT / µ-CT / nano-CT) or electrons in the transmission electron microscope (TEM), is to reconstruct the true object data out of a series of sampling measurements (projections) as accurately as possible, as illustrated in Figure 1.

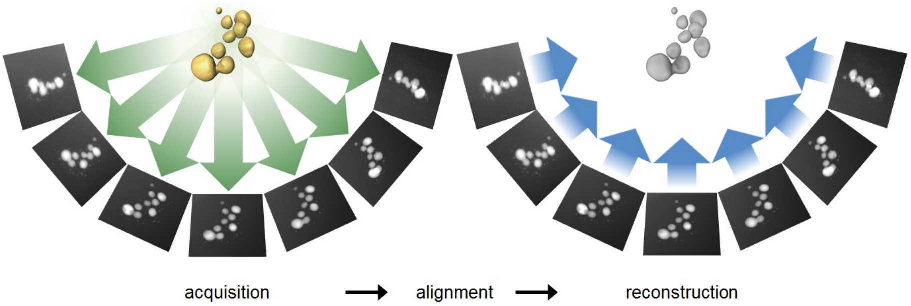

Abbildung 1: Principle of computed tomography (here, electron tomography): a tilt series of projections from various viewing angles is acquired and, after its alignment, subsequently used to reconstruct the object data (here, exemplarily demonstrated for gold nanoparticles). [5]

The seminal works of Candès, Romberg, Tao [3] and Donoho [2] proved that given additional information about the sparsity of the original signal, here the object data, improves the recovery accuracy significantly. The goal of the present thesis is to impose further specific information in order to improve the accuracy of the observed 3D reconstruction even further (cf. Figure 2c). This is expecially of interest, if the data sampling is reduced, e.g., angular undersampling leads to strong streaking artifacts in the 3D reconstruction, or if the projection data is incomplete, i.e., projections taken within a tilt-angle range range <180° leads to so-called missing-wedge artifacts in the reconstruction (cf. Figure 2a,b).

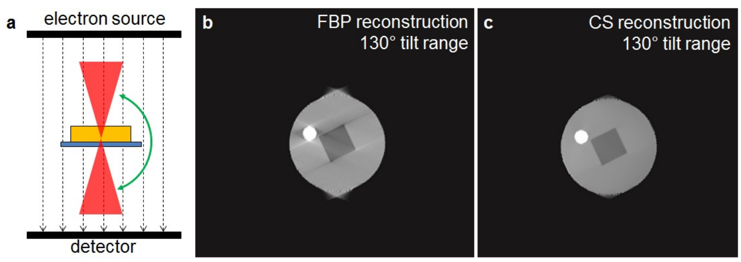

Abbildung 2: (a) Sketch of the situation of a limited tilt–angle range (green arrow) in the transmission electron microscope causing a missing wedge (marked in red). (b) Standard filtered backprojection (FBP) reconstruction of a 2D test object slice with a typical tilt–angle range of 130° (missing wedge of 50°) for conventional electron tomography (ET) and 1° tilt increment between the projections. (c) Respective compressed sensing (CS) reconstruction (here, implying sparsity through total variation minimization) with the same amount of projections as in (b) – a reduction of reconstruction artifacts (streaking and missing–wedge artifacts) is visible. [5]

Consider a coefficient matrix $R \in \mathbb{R}^{m \times n}$, where $n = 2, 3$ already appears in interesting applications (so–called Radon and X–ray transform), and a potentially uncertain output signal $p(u) \in \mathbb{R}^n$ (measured projection data) with predefined uncertainty set $\mathscr{U}$. The vector $f \in \mathbb{R}^n$ is shall resemble the true object data, that has to be reconstructed. If we further impose sparsity on f in a sense that at most $k \in \mathbb{R}$ columns of R explain the output signal properly, we obtain the following nonlinear optimization problem:

$$\min_{f \in \mathbb{R}^n} \max_{u \in \mathscr{U}} \|p(u) - Rf\|_2^2 \tag{1a}$$

$$\text{s.t. } \|f\|_1 \leq k, \tag{1b}$$

$$f_1 \cdots f_n = 0, \tag{1c}$$

$$f_i \geq 0. \tag{1d}$$

Observe, that constraint (1c) adds additional information to the standard formulation used in compressed sensing. In general, (1) is a non-convex problem that is NP-hard to solve. However, for special instances one may convexify the above problem in order to achieve tractable reformulations.

Specific questions and tasks:

1. Literature review: Revisit the model formulation of (1) and provide an overview on the existing literature on compressed sensing and its applications to computed tomography (X–ray and electron tomography [1, 4]).
2. Linearize constraint (1c) in order to perform numerical studies on test instances in the case $p(u) = p$, i.e., with no uncertainties involved.
3. Is the developed approach capable to deal with uncertainties?

Supervision

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Literatur

[1] K. J. Batenburg und J. Sijbers. “DART: A Practical Reconstruction Algorithm for Discrete Tomography”. In: *IEEE Transactions on Image Processing* 20.9 (2011), S. 2542–2553. DOI: 10.1109/TIP.2011.2131661.

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